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## LIST OF SYMBOLS

<b>K</b>	-	Stiffness matrix
<b>B</b>	-	Matrix differential operator for shape function
<b>D</b>	-	Constitutive matrix
<b>f<sub>b</sub></b>	-	Boundary vector
<b>f<sub>l</sub></b>	-	Load vector
<b>f<sub>0</sub></b>	-	Initial strain vector
<b>N</b>	-	Shape function
<b>ε<sub>0</sub></b>	-	Initial strain
<b>σ</b>	-	Stress
<b>ε</b>	-	Strain
<b>α</b>	-	Thermal expansion coefficient
<b>ΔT</b>	-	Change in temperature
<b>°C</b>	-	Temperature
<b>t</b>	-	Traction vector
<b>n</b>	-	Normal vector
<b>b</b>	-	Body force
<b>E</b>	-	Young's modulus
<b>ν</b>	-	Poisson's ratio
<b>u</b>	-	Displacement vector

## **GLOSSARY OF TERMS**

FEM	-	Finite Element Method
FDM	-	Finite Difference Method
BEM	-	Boundary Element Method
dof	-	Degree of freedom

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